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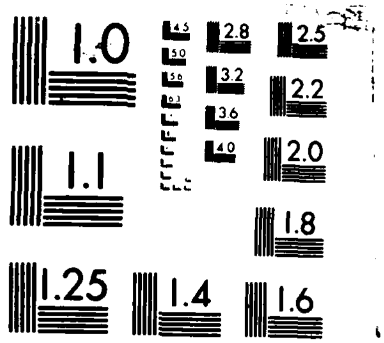

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Probablistic Analysis of Semilinear Partial Differential Equation

Joseph Glover, Principal Investigator

Kai Lai Chung, Co-Principal Investigator

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4. Symmetric Markov Processes and Capacities.

In collaboration with M. Rao and W. Hansen, Glover showed that the electrostatic capacity of a symmetric potential theory determines the potential theory uniquely. One consequence of this result is that two symmetric Markov processes with the same capacities must be time changes of one another.

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There are several concrete results of importance which have emerged from our research in probability and differential equations. There are also several others which may turn out to be important.

1. Systems of Semilinear Elliptic Equations:

The first is the discovery of an algorithm for solving systems of semilinear elliptic and parabolic partial differential equations such as

$$-\Delta u_1 + f(u_1,u_2) = g$$
$$-\Delta u_2 + h(u_1,u_2) = k \ .$$

Such equations show up throughout all of engineering, physics, chemistry and biology, and there is an enormous amount of work currently being done to understand these equations and the existence and behavior of their solutions. Glover has been fortunate enough to discover an iterative method which constructs a solution to this system under certain hypotheses. These hypotheses are verifiable in many circumstances and actually seem to be quite generally applicable. From a practical standpoint, his discovery allows scientists to construct solutions to these equations iteratively on a computer, something which they could not do easily before. The previously available algorithm was based on a generalization of the monotone method of sub- and super-solutions for *one* equation. This monotone method generalizes only to special systems in which the nonlinearities have a particular form, while ours goes way beyond this and allows us to solve general systems. From a theoretical standpoint, our approach permits us to give a unified treatment of elliptic, parabolic and integro-differential semilinear systems and makes clear that solving such equations is best understood as a problem in potential theory. While the algorithm and the result may be described purely in terms of differential equations, probabilistic techniques are crucial to prove the algorithm works. (For a detailed description of the algorithm, see the Research Progress and Forecast Report of April 21, 1986.)

2. Gauge and Conditional Gauge Theorem.

Chung is continuing his recent work in gauge theory for solving Schrodinger equations probabilistically. A principal tool is the Green function in R^d. Chung has obtained a new sharp estimate for the Green function in a ball. If $d \geq 3$ and B is an open ball, let $p(x)$ be the distance from x to the boundary of B, ∂B, and let

$$F(x,y) = \min \{|x-y|^{2-d}, p(x)p(y)|x-y|^{-d}\} .$$

If G is the Green function for B, then there are constants c_1 and c_2 depending on d so that $c_1 F \leq G \leq c_2 F$.

For d=2, if the radius of B is less than 1/2 and if $F(x,y) =$

min $\{\log|x-y|^{-1}, p(x)p(y)|x-y|^{-2}\}$, then $G \leq cF$. An example shows that the reverse inequality with another constant c is false. For $d = 2$, it is not known if there is a sharp estimate with a different F.

Recently, in collaboration with M. Rao, K. L. Chung has examined the theory of Schrodinger equations of the form $-\Delta u + qu = f$ from the Markov process point of view. Many of the recent results about these equations involve functions q which may be quite large and which may take both positive and negative values. (This situation turns out to be especially important in physics.) Until recently, it seemed that many of the results depended crucially on the Laplacian Δ being in the equation and lending its special character to the solution of the problem. Chung and Rao have used Markov processes to show that most of the results are simply general facts about infinitesimal generators of semigroups (the Laplacian being one such). This discovery is revising peoples' approach to the subject. It allows them to carry over many results obtained through hard analysis of the Laplacian to situations where the Laplacian is replaced with an elliptic, parabolic or integro-differential operator.

3. Markov Processes with Random Times of Birth and Death.

In collaboration with R. K. Getoor, Glover recently wrote an article extending and simplifying a theorem of Kuznetsov allowing construction of Markov processes with random times of birth and death. Such processes arise in linear Markov process theory as well as in nonlinear situations such as $\partial u/\partial t = \Delta u + h(u)$.

4. Symmetric Markov Processes and Capacities.

In collaboration with M. Rao and W. Hansen, Glover showed that the electrostatic capacity of a symmetric potential theory determines the potential theory uniquely. One consequence of this result is that two symmetric Markov processes with the same capacities must be time changes of one another.

5. Directions of Research.

Glover intends to continue his work on systems of semilinear elliptic equations. The theoretical procedure he discovered relies on transfinite induction, which cannot be implemented on a computer. But computers have only finite accuracy, anyway, and it is possible to develop an algorithm which will pinpoint the solution to any desired accuracy. He also intends to continue investigating the hypotheses necessary for the procedure to work and their relationships.

Growing out of the work described above has come another result which may prove to be of practical importance in the design of flexible structures. In joint work with P. J. McKenna, Glover has simulated a bridge suspended by one cable, but taking into account the fact that the restoring forces are different if the bridge bed is displaced upwards or downwards. Our numerical results are rather striking. For appropriate choices of the parameters in the equation, we obtain the following type of behavior. Subject the bridge to an oscillation of magnitude b (such as might be produced by a brisk wind). The resulting steady state oscillation of the bridge is of similar magnitude. But if one increases the magnitude of the oscillation by a very small number (in our simulations, it is on the order of one ten-thousandth), the magnitude of the oscillation of the bridge increases by a factor of five to ten ! This is a direct consequence of the fact that the equation modelling this phenomenon is nonlinear, and we intend to investigate this further.

Chung intends to continue his work on Gauge theory, Schrodinger equations and nonlinear equations.

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